Why do we observe a weak force? The hierarchy problem in the multiverse

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Unless the scale of electroweak symmetry breaking is stabilized dynamically, most of the universes in a multiverse theory will lack an observable weak nuclear interaction. Such "weakless universes" could support intelligent life based on organic chemistry, as long as other parameters are properly adjusted. By taking into account the seemingly-unrelated flavor dynamics that address the hierarchy of quark masses and mixings, we show that such weakless (but hospitable) universes can be far more common than universes like ours. The gauge hierarchy problem therefore calls for a dynamical (rather than anthropic) solution.

Bayesian statistics for multiverse. The common wisdom is that, due to the vast landscape of configurations that are local energy minima, string theory does not uniquely predict the spectrum of particles and interactions as observed in our Universe [1]. Eternal inflation might then generate an enormous number of causallydisconnected "pocket universes," each with its own laws of physics [2]. It is unlikely that we shall ever study directly the physics of universes other than our own, but a limited Bayesian analysis is still possible in such a scenario [3]: Let $\{\alpha_i\}$ be the set of parameters of the Standard Model (SM), and let $\{\alpha_i^{\oplus}\}$ represent the values that we have deduced from experiments in our own Universe. The theory describing the multiverse (whether it be string theory or something else) in principle gives a probability density function (PDF), $p(\{\alpha_i\})$, corresponding to the fraction of universes with those parameters. By Bayes's theorem, the probability of measuring a set of parameters $\{\alpha_i\}$ is

$$\begin{split} p_{\text{measured}}(\{\alpha_i\}) &\equiv p(\{\alpha_i\}|\text{observer}) \\ &\propto p(\text{observer}|\{\alpha_i\}) \cdot p(\{\alpha_i\}) \;. \end{split} \tag{1}$$

The conditional PDF for an observer existing, given a set of values for the SM parameters, $p(\text{observer}|\{\alpha_i\})$, is the anthropic factor in this analysis. All that we may say with definiteness is that, if

$$r \equiv \frac{p_{\text{measured}}\{\alpha_i^{\oplus}\}}{p_{\text{measured}}\{\alpha_i^{*}\}} \ll 1 , \qquad (2)$$

where $\{\alpha_i^*\}$ is a set of parameters well outside the tolerances of the observed $\{\alpha_i^{\oplus}\}$, then the theory makes our own "way of life" an unexplained rare event.

Hierarchies. In the SM, M_{weak} is given by

$$M_{\text{weak}} = \frac{1}{2}gv, \qquad (3)$$

where g is the weak coupling constant and v is the vacuum expectation value (VEV) of the Higgs field. Quantum corrections to (mass)² of the Higgs depend quadratically on the cutoff scale, which one would expect to be

of the order of $M_{\rm Pl}$ (the Planck mass), at which quantum gravitational effects become dominant. As Wilson first stressed [4], this implies that the observed hierarchy

$$\frac{M_{\text{weak}}}{M_{\text{Pl}}} \sim 10^{-16}$$
 (4)

requires a fine-tuning of the bare (mass)² of the Higgs field, to a part in 10³². A vast amount of work has gone into proposing theoretical models, such as technicolor [5] and supersymmetry [6], that invoke still undetected interactions in order to obtain the "gauge hierarchy" of Eq. (4) without fine-tuning.

An even greater hierarchy appears in cosmology: the "dark energy" scale $M_{\rm DE}$, deduced from the acceleration of the rate of expansion of the Universe [7], is

$$\frac{M_{\rm DE}}{M_{\rm Pl}} \sim 10^{-31} \ .$$
 (5)

Weinberg argued in [8] that the smallness of $M_{\rm DE}/M_{\rm Pl}$ might have an anthropic explanation: only very small values of $M_{\rm DE}$ are observable, since otherwise the dark energy would prevent structure formation in the Universe, and could therefore be incompatible with the presence of any conscious observer. In other words, in Eq. (1), $p({\rm observer}|\{\alpha_i\})$ is zero unless the absolute value of $M_{\rm DE}$ is very small compared to $M_{\rm Pl}$. If very small values $M_{\rm DE}/M_{\rm Pl}$ are at all possible (i.e, if $p(\{\alpha_i\})$ is non-zero for such values), it might not require deliberate fine-tuning to explain why we observe a dark-energy hierarchy.

Does the gauge hierarchy of Eq. (4) also admit an anthropic explanation? If all other SM parameters are held fixed, then v could not be more than about 5 times greater without destabilizing atoms [9], and such conditions might well be incompatible with intelligent observers. This argument, however, relies on scanning v while fixing other SM parameters at their observed values. Whether this assumption is justified, and what the consequences of relaxing it are, will be our focus here.

In the SM, a fermion mass m_f depends on both the Higgs VEV v and the Yukawa coupling y_f :

$$m_f = \frac{1}{\sqrt{2}} y_f v \,. \tag{6}$$

Fermions in the SM are organized into generations ("flavors"), identical except for their Yukawa couplings. The SM flavor sector is described by 13 parameters, plagued by hierarchies that constitute the so-called "flavor puzzle." For instance, one would generically expect $C^{\rm SM}$, the "Jarlskog determinant" that characterizes the amount of charge-parity violation in the SM, to be ~ 0.1 , whereas in reality $C^{\rm SM} \sim 10^{-22}$ [10] (far too small to explain our Universe's matter-antimatter asymmetry).

Except for rare, high-energy processes, most of the physical phenomena in our Universe depend on the masses of only the three lightest fermions: the electron e, and the u and d quarks. One might therefore expect that only these masses would play an important role in anthropic constraints on possible laws of physics. Thus, the flavor puzzle does not seem to admit an anthropic solution, calling instead for a dynamical solution to account for the observed hierarchies. Remarkably, we shall see that by incorporating such flavor dynamics —which has no known theoretical connection to the weak scale—both v and the y_f 's are likely to scan over the multiverse, and that stable atoms become possible (even favored!) with $v \gg v^{\oplus}$.

Weakless universe. Before analyzing the implications of flavor dynamics in the multiverse, let us briefly summarize the argument of [11], as to how to build a hospitable universe without weak interactions. The SM parameters of this universe are indicated by a superscript *. Let

$$v^* \simeq M_{\rm Pl}$$
 (7)

(i.e., the Higgs VEV takes its most natural value). The gauge groups and low-energy gauge couplings are the same as in our Universe and $\Lambda_{\rm QCD}^* = \Lambda_{\rm QCD}^{\oplus}$ (where $\Lambda_{\rm QCD}$ is the energy scale below which the strong nuclear interaction is non-perturbative). The Yukawa couplings are:

$$y_f^* = \frac{y_f^{\oplus} v^{\oplus}}{v^*}, \text{ for } f = e, u, d$$
 (8)

(i.e., $m_f^* = m_f^{\oplus}$, for f = e, u, d). As long as

$$m_s \gtrsim \frac{m_u + m_d}{2} + 5 \text{ MeV},$$
 (9)

the s quark will not participate in forming stable nuclei (which otherwise could have no more than a couple of units of electric charge, leading to incompatibility with organic chemistry) [12]. For simplicity, we just require that

$$m_f^* \gtrsim \Lambda_{\rm QCD}^*$$
 (10)

for all flavors other than e, u, and d.

In the weakless universe, the usual fusing of four protons to form helium would be impossible, because it requires that two of the protons convert into neutrons, via the weak force. But other pathways could exist for nucleosynthesis: A small adjustment to the parameter that characterizes the matter-antimatter asymmetry of the Universe, $\eta_b^* \simeq 10^{-2} \eta_b^{\oplus}$, (where $\eta_b^{\oplus} \sim 10^{-10}$ is the cosmological baryon-to-photon ratio) is enough to ensure that the Big Bang nucleosynthesis would leave behind a substantial amount of deuterium nuclei. Stars could then shine by fusing a proton and a deuterium nucleus to make a helium-3 nucleus [11]. Such a universe could still produce heavy elements up to iron, stars might be able to shine for several billion years at an order-one fraction of the brightness of stars in our own Universe, and type Ia supernova explosions could still disperse the heavy elements into the interstellar medium.

In terms of the Bayesian analysis of Eq. (1), it therefore seems that

$$p(\text{observer}|\{\alpha_i^{\oplus}\}) \simeq p(\text{observer}|\{\alpha_i^*\})$$
. (11)

Naturalness. The dependence of v on quantum corrections to the (mass)² parameter for the Higgs field suggests that large values of it are more likely. Since the sign of v is not physical, the most conservative assumption is

$$p(v) \sim \frac{v^2}{M_{\rm Pl}^2}$$
 for $0 < v < M_{\rm Pl}$. (12)

Another way of stating the gauge hierarchy problem is that Eq. (12) suppresses the likelihood of finding a universe with a Higgs VEV as small as v^{\oplus} , by a factor of 10^{-32} . It is unclear at this point how v might be distributed in the landscape of string theory [13, 14], but Eq. (12) agrees with the expectation from the toy landscape considered in [15], if no symmetry requires the mean value of v to vanish. In any case, we shall argue that unless the PDF for v is significantly less biased toward large values than Eq. (12), then a multiverse theory could fail the Bayesian test of Eq. (2).

Flavor dynamics. During the past 30 years or so, theoretical physicists have proposed various mechanisms that might explain the flavor hierarchy puzzle [16–18]. The resulting wisdom regarding the generation of flavor hierarchies can be summarized by the following relation between the fundamental flavor parameters and the effective Yukawa couplings:

$$y \propto \epsilon^Q$$
 , (13)

¹ This is usually called a "puzzle," rather than a "problem," because the flavor parameters are not subject to large additive quantum corrections.

where ϵ is some small universal parameter and Q is a flavor-dependent charge. (In these models, the values of y are usually related to the VEV of a scalar field.) Consider a general power-law distribution for the charge Q on the multiverse,

$$p(Q) \propto Q^n$$
 , (14)

and let ϵ have an arbitrary distribution $p(\epsilon)$. Then the PDF for y is

$$p(y) \propto \int d\epsilon \, dQ \, p(\epsilon) Q^n \delta(y - \epsilon^Q)$$

$$= \int d\epsilon \, dQ \, p(\epsilon) Q^n \frac{\delta(Q - \ln y / \ln \epsilon)}{y \ln \epsilon}$$

$$= \left(\frac{\ln^n y}{y}\right) \times \int d\epsilon \frac{p(\epsilon)}{\ln^{n+1} \epsilon} \propto \frac{\ln^n y}{y} . (15)$$

Let us evaluate the r of Eq. (2) as the ratio of hospitable universes with $v=v^{\oplus}$ (i.e., weak universes, like our own) and $v=v^*\sim M_{\rm Pl}$ (i.e., weakless universes), with all other parameters integrated over. This r has two factors: the first is the ratio of probabilities for the corresponding values of v, which by Eq. (12) is simply $(v^{\oplus}/M_{\rm Pl})^2$. The second is the ratio of the probabilities for the Yukawa couplings that produce the corresponding quark masses (see Eq. (6)).

By Eq. (15), each light quark (i.e., each quark with a mass below $\Lambda_{\rm QCD}$) contributes to r a factor of

$$\frac{\int_0^{\Lambda_{\rm QCD}/v^{\oplus}} dy \, p(y)}{\int_0^{\Lambda_{\rm QCD}/M_{\rm Pl}} dy \, p(y)} = \frac{\left[\ln^{n+1} y\right]_0^{\Lambda_{\rm QCD}/v^{\oplus}}}{\left[\ln^{n+1} y\right]_0^{\Lambda_{\rm QCD}/M_{\rm Pl}}} \,, \tag{16}$$

which is equal to 1 for $n \ge -1$, or to

$$\left[\frac{\ln \left(\frac{\Lambda_{\text{QCD}}}{M_{\text{Pl}}} \right)}{\ln \left(\frac{\Lambda_{\text{QCD}}}{v^{\oplus}} \right)} \right]^{-n-1} \approx 6.7^{-n-1}$$
(17)

for n < -1. On the other hand, for a single heavy quark, the corresponding factor is less than or equal to 1, for any n.

Thus, the full ratio r of weak to weakless universes with two light quarks —which are necessary to get organic chemistry [12]— and any number of heavy quarks, can be bounded by

$$r < \left(\frac{v^{\oplus}}{M_{\rm Pl}}\right)^2 \times \left\{ \begin{bmatrix} \frac{\ln\left(\frac{\Lambda_{\rm QCD}}{M_{\rm Pl}}\right)}{\ln\left(\frac{\Lambda_{\rm QCD}}{v^{\oplus}}\right)} \end{bmatrix}^{-2n-2} & n < -1, \\ 1 & n \ge -1. \end{bmatrix}$$
 (18)

We therefore conclude that $r \ll 1$ for any $n \gtrsim -21$. Unless ϵ were made unnaturally small (which would be a hierarchy problem of its own), a model with n < -21 would

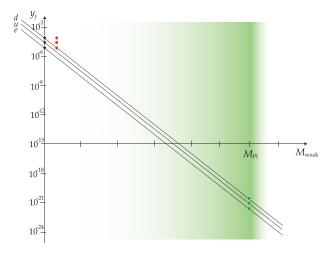


FIG. 1. Log-log plot of the Yukawa couplings y_f as functions of $M_{\rm weak}$. Black stars correspond to our Universe, red hexagons to a spectrum incompatible with stable atoms [9] and green squares to the weakless universe described by Eqs. (7) and (8). The green shading roughly indicates the expected probability density in the multiverse, from Eqs. (12) and (19).

strongly favor large Yukawa couplings and would conflict with the observed distribution of fermion masses.²

For n = 0, Eq. (15) becomes

$$p(y) \propto \frac{1}{y}$$
, (19)

which corresponds to a scale-invariant distribution, in which y is uniformly distributed on a logarithmic scale.³ Approximate scale-invariance (between cutoffs) is a common feature of complex dynamics and leads, for instance to "Benford's law," the well-known statistical observation that the first digits of data from a surprisingly wide variety of sources are logarithmically distributed, with the digit 1 being six times more likely than the digit 9 [20]. Approximate scale-invariance is therefore a plausible expectation for something like the distribution of Yukawa couplings in the multiverse, independently of specific flavor models. Figure 1 illustrates why, if the Yukawa couplings have a scale-invariant distribution, Eq. (12) would imply that we would be far more likely to find ourselves living in a weakless universe, rather than in one with a measurable weak nuclear interaction.

The authors of [21] consider a PDF for y of the form of Eq. (19), which requires cutoffs. A lower cutoff can disallow the $y_f^* \sim 10^{-21}$ of the weakless universe. There

² The anthropic requirement that the electron be light [14, 19] can be accommodated in this analysis, without significantly altering the conclusions.

³ It is easy to show that —other than $\delta(x)$ — 1/x is the only scale-invariant PDF, but it is not normalizable, and needs to be cut off at some minimum and maximum values.

is no known dynamical reason, however, why such values would be forbidden in the string landscape.⁴ Our results, expressed in Eqs. (16) and (18), are cutoff independent, and differ from those of [21] regardless of the PDF used for the Higgs VEV.

Discussion. The gauge hierarchy problem of Eq. (4) has led to an enormous effort of "model-building," i.e., of proposing new dynamical symmetries that might explain the smallness of $M_{\text{weak}}/M_{\text{Pl}}$. But the inflationary multiverse and the string landscape raise the disturbing possibility that such efforts could be as misguided as Kepler's attempts, in the 16th century, to explain the Titius-Bode law (an approximate mathematical regularity in the sizes of the orbits in our solar system) in of terms nested Platonic solids [23]. We now know that the laws of gravity are compatible with a huge variety of solar systems and that the only significant constraints on the details of our own solar system, such as the sizes of orbits, are anthropic.

Proposed anthropic explanations of the observed values of the parameters of the SM, however, should take into account that many parameters could scan over the multiverse and that the variation of one might compensate for the variation of another in such a way that life could be possible in universes different from ours. The flavor hierarchy puzzle (which a priori has nothing to do with the weak scale) suggests that the Yukawa couplings of the SM may be realized as VEVs of dynamical fields [16–18]. If that is the case, then the Yukawa couplings, though dimensionless in themselves, may scan on the multiverse along with the dimensionful quantities [15], and fairly generic considerations about flavor dynamics lead us to expect that most hospitable universes would lack an observable weak force. This presents a serious challenge to the notion that the gauge hierarchy problem can be solved anthropically.

Unless the probability density function of the SM parameters in the multiverse turns out to have some quite specific features (such as a strong bias towards small values of the Higgs self-coupling [24]), the solution to the gauge hierarchy problem should be *dynamical*, and some new, detectable interactions —beyond those of an elementary Higgs field— should appear in this Universe at energy scales currently being probed by the Large Hadron Collider. The multiverse does not do away with the need for model-building.

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⁴ In our Universe, the uncertainties in the extraction of the value of the light quark masses are such that $y_u^{\oplus} = 0$ is less than 3 standard deviations away from the central experimental value [22].

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